# Direct and Cross-Polarized Backscatter Radar Cross Sections of a Turbulent Plasma

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## Theme

IN this paper, the backscattered RCS (radar cross section) from a turbulent plasma is calculated. The computation is restricted to a single scattering in the plasma. However, no total internal reflection of the incident radiation gives rise to a depolarization of the backscattered return. The RCS is calculated as a function of the size and electron density of the plasma, the frequency, and the incident angle of the radiation.

## Contents

The Lau and Watson[1] vector radiative transport model is used to study scattering in an underdense, inhomogeneous, turbulent plasma when refractive effects are significant. The radiation is incident at an angle $\alpha$ on an infinitely long plasma cylinder of a square cross section, width $W$ (Fig. 1), which has an electron density ($n_e$) that is constant for any square centered on the axis but which varies uniformly from square to square. A calculation is made of the backscattered RCS for electromagnetic waves that are scattered once in the plasma. Attenuation of the signal by scattering other than backscatter is included. The authors[2,3] have shown previously that in a plasma when there is refractive scattering, the presence of an alternate path gives rise to a cross-polarized RCS, even though multiple scattering is ignored, and enhances the direct-polarized RCS. Earlier investigators [4,5] have used only multiple scattering theories to predict the cross-polarized RCS.

This study was performed with a circularly polarized incident wave. The results are quite similar, but not identical, to those obtained for a linearly polarized wave. In the case of a linear wave, consider an arbitrarily polarized incident signal decomposed into electric field vectors normal and parallel to the plane of Fig. 1. If the ray is scattered (but in the plane of the incident ray) into an angle $\theta$ with respect to the unscattered wave, the normal component will be unaffected by the scattering. The parallel component will be foreshortened by the factor $\cos 2\theta$. Comparing the depolarizing backscattered ray to the normal backscattered ray, we find that the normal components are identical, but the parallel components differ in magnitude: hence the depolarization of the second ray.

The geometric optics model of refraction has been used in this analysis. That is, the path of the refracting electromagnetic signal is traced and the backscattered signal is summed over all rays. Unlike the situation in a circular cylinder, there is no bending in the direction of $y$, transverse to the plane formed by the axis and that incident ray which would intersect the axis if projected in a straight line. To obtain the total direct- and cross-polarized signals, scattering along the complete ray path in the direction of the two alternate paths must also be summed. Attenuation of the signal is counted along the entire path of the scattered beam.

The assumed electron density distribution for the calculation is Gaussian in the central plasma region in both directions and becomes linear in the outer region. The mean value of electron density is assumed to be three times the rms value in this analysis. In a typical plasma, the relative level of fluctuations has a wide range of values, and the ratio chosen for this analyais is somewhat arbitrary. It is also assumed that this plasma turbulence is isotropic and that the measurements of the plasma RCS in units of square meters/meter will be made in the inertial subrange of the fluctuation-dissipation spectrum. The von Karman interpolattion formula is used to describe this spectrum, and the correlation length was obtained in Ref. 6.

In Fig. 2 is shown a plot of the calculated direct RCS from the direct path [$\sigma$DP($s$)] the total direct RCS from both paths ($\sigma_{\mathrm{DP}}$), and the cross-polarized RCS ($\sigma_{CP}$) vs the mean axial electron density. The dependence of the direct-polarized RCS before refraction on the fluctuating electron density goes as the square, as predicted by the Born approximation. If part of the beam is internally reflected, the cross section is no longer a strong function of $\bar{n}_e$. In fact, the direct RCS is essentially constant. However, the extra signal return generated along both alternate paths does cause the total direct signal to

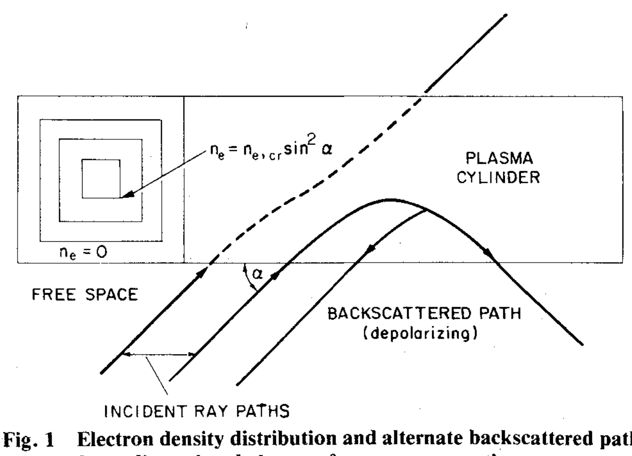


**Fig. 1 Electron density distribution and alternate backscattered path length of two-dimensional plasma of square cross section.**

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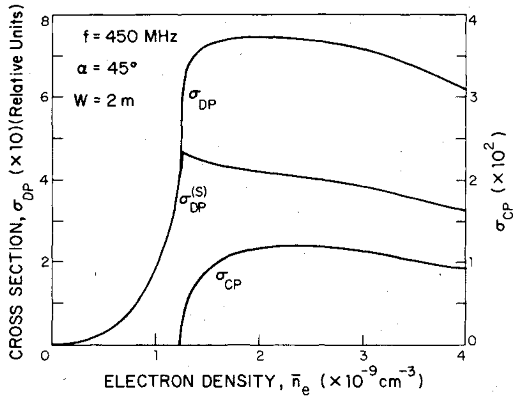


**Fig. 2 RCS as a function of electron density.**

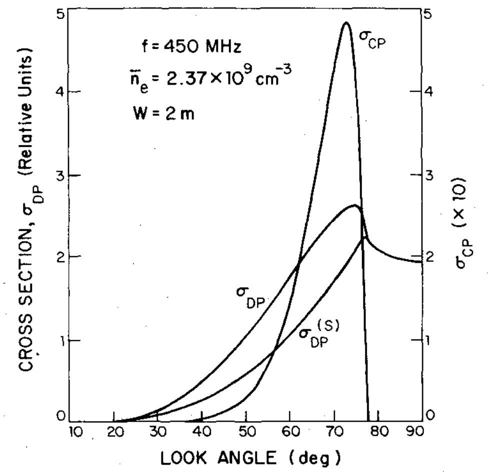


**Fig. 3 RCS as a function of look angle.**

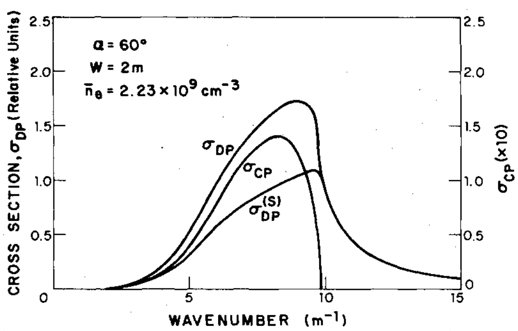


**Fig. 4 RCS as a function of wave number.**

rise to about twice the level of RCS calculated for just the backscattered path. The cross-polarized signal is small and reaches a constant value at about the value of electron density at which the total direct-polarized RCS becomes constant.

The cross-polarized RCS is not small in Fig. 3. Provided that the incident angle is less than the critical angle for internal reflection, the cross-polarized return will increase rapidly with $\alpha$. The cross-polarized signal will be a very large fraction of the total backscattered return if the look angle is greater than 85°.

The frequency dependence of the plasma RCS is shown in Fig. 4. the maximum value of each backscattered RCS is attained at a frequency somewhat higher than the frequency at which the product of correlation length and wave number is one.

The decrease in RCS with increasing frequency is due to the very strong functional dependence on frequency of the von Karman spectral function. Also, the distance from the axis at which total internal reflection occurs decreases as frequency increases, with reflection not occurring at any distance when

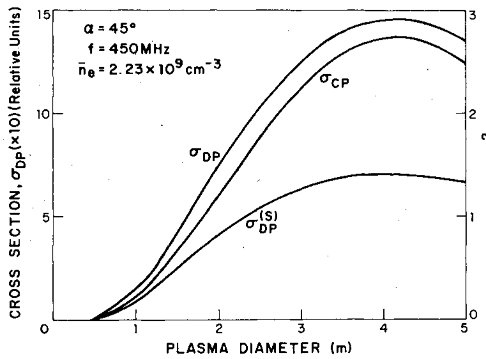


**Fig. 5 RCS as a function of plasma diameter.**

$$f > (c/\sin\alpha)\ (n_e(z=0)r_0/\pi)^{1/2} \tag{1}$$

where $r_0$ is the classical electron radius. All the RCS decrease as the frequency decreases when $f$ is small because the signal penetrates only a small region of the plasma. For very small values of $kW$, the geometric optics model breaks down and a more complete model of refraction is necessary to describe the scattering.

The dependence of the RCS on plasma diameter is illustrated in Fig. 5. The dependence on $W$ is considerably different from the effect that a change in the electron density has on the RCS. The initial increase in RCS comes about because of the increase of the plasma surface area facing the beam and of the distance to the turning point. This is eventually more than offset by the decrease in the relative values of the attenuation length and the spectral function. Also, the distance to the position of total internal reflection ceases to be important because of the large amount of attenuation in the plasma.

If attenuation is ignored in the analysis by assuming that the attenuation length is infinitely long, then some of the calculations of RCS are quite different. A comparison[3] indicates that as expected, the direct-polarized RCS grows faster if the signal is unattenuated until the electron density is so large in the plasma that refraction occurs. Once refraction occurs, the RCS show the same behavior as a function of electron density as was exhibited when attenuation was included.

It may be concluded, then, that the cross-polarized backscattered RCS based on calcualtions using a geometric optics model, in a plasma in which the multiple scattering cross-polarized signal return has been ignored, is small except when the beam enters nearly normal to the axis. Using multiple scattering effects Ruffine and DeWolf[4] have shown in their calculations a far greater contribution to the cross-polarized RCS at small look angles than has been shown in this analysis. On the other hand, except at high look angles, the "extra path" contribution to the direct-polarized backscattered signal is significant.

## References

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